Network Flow

How to solve maximal flow and minimal cut problems

What is Network Flow?

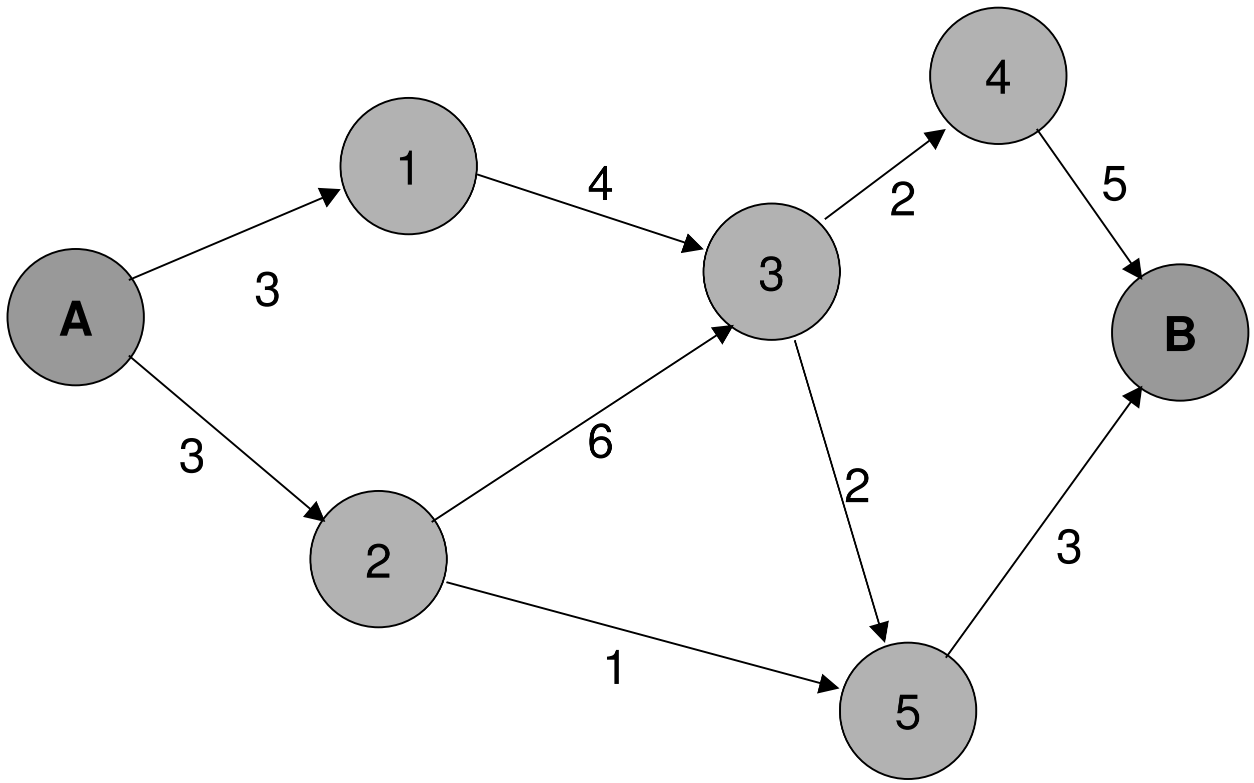
- Network Flow is a subsection of graph theory related specifically to situations where something moves from one location to another.
- An easily visualized example of network flow is piping system through which a quantity of water must pass.

Concepts in Network Flow

- “Sources” are nodes which supply a commodity
- “Sinks” are nodes which use up a commodity
- An edge’s capacity is the maximum amount of flow which can pass through it
- Graphs are usually directed

An example

- What is the maximum flow from A to B?



The Ford-Fulkerson method

- This is a relatively simple way to find maximum flow through a network:
  - Find an unsaturated path from the source to the sink
  - Add an amount of flow to each edge in that path equal to the smallest capacity in it
  - Repeat this process till no more paths can be found
  - The total amount of flow added is then maximal

Storing the graph

- It is easiest to store, for each edge in the graph, its capacity in both directions, as well as the current flow in each direction.
- Thus for a directed graph, the capacity in the reverse direction will start as zero.
- When flow is put through the edge, decrease its capacity in the direction and increase the capacity in the opposite direction.

Finding a path

- The eventual answer given by this method is not affected by the path chosen, although the algorithm's speed will be affected
- If possible, the shortest path or the path with maximum flow is a good choice, else simply using a DFS works reasonably well

Minimum Cuts

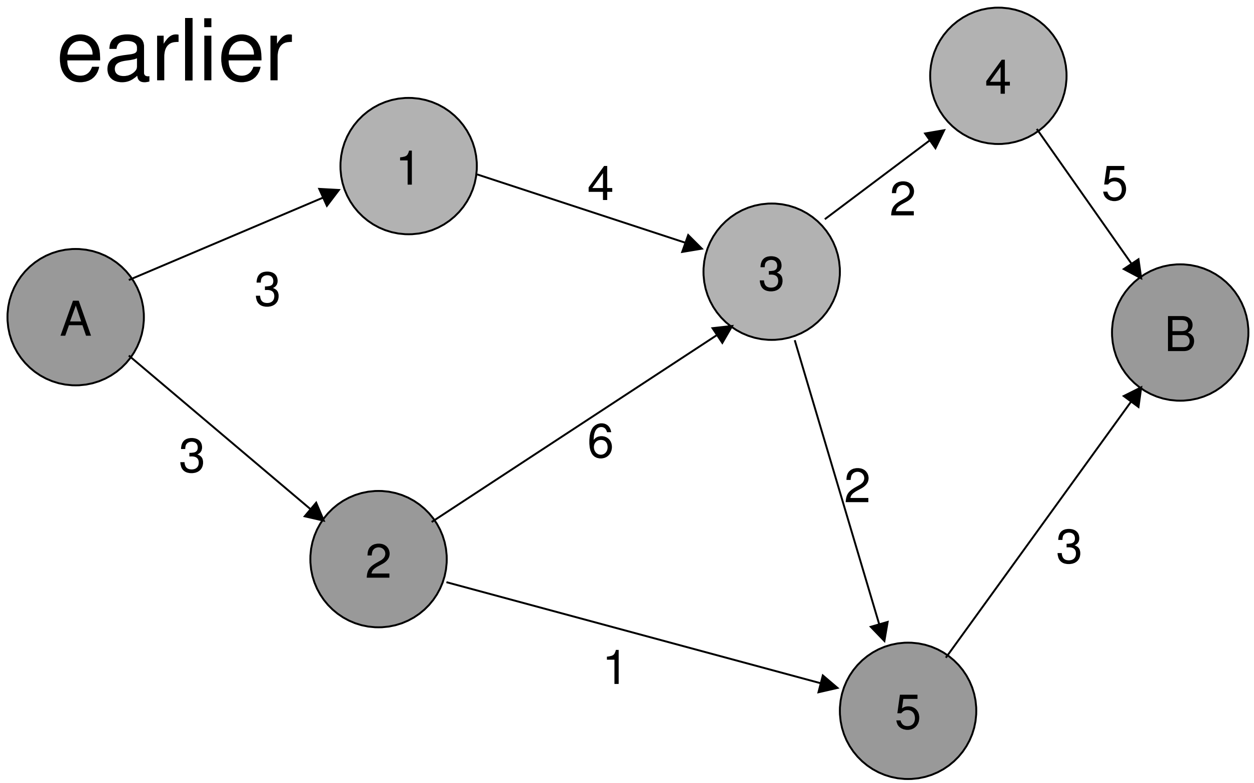
- It is sometimes necessary to know the minimum total weight of a cut that splits the graph in two, separating the source and the sink
- This will be equal to the maximum flow through the network

To find the minimum cut

- First create the maximum flow graph
- Select all the nodes that can be reached from the source by unsaturated edges
- Cut all the edges that connect these nodes to the rest of the nodes in the graph
- This cut will be minimal

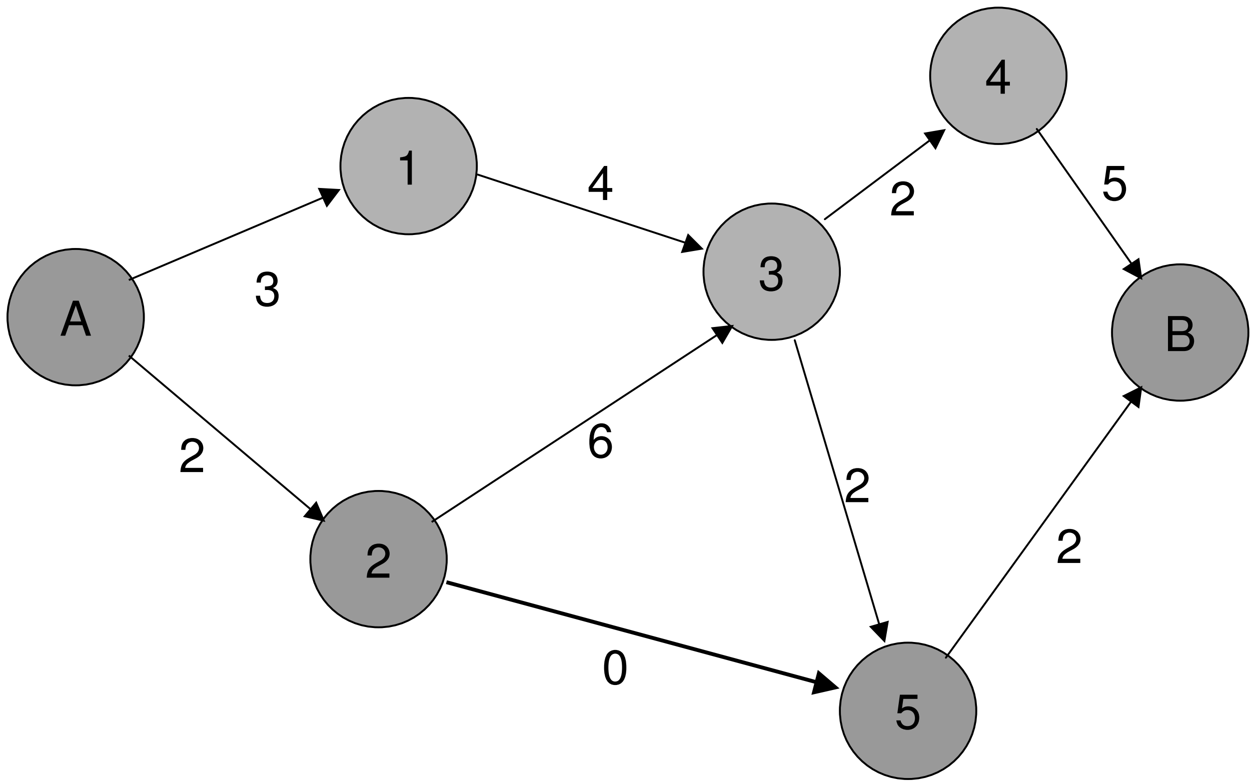
An example

- Maximal flow in the graph shown earlier



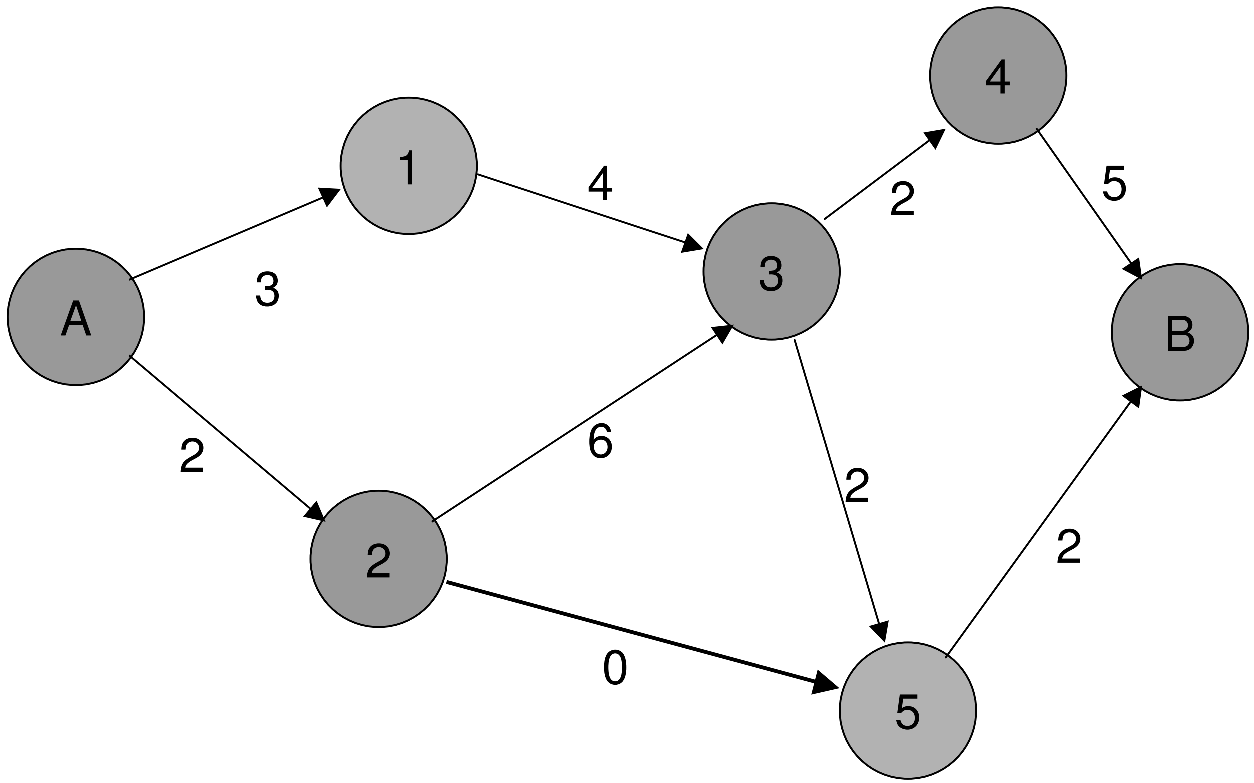
An example

- Choose a path and put flow through it



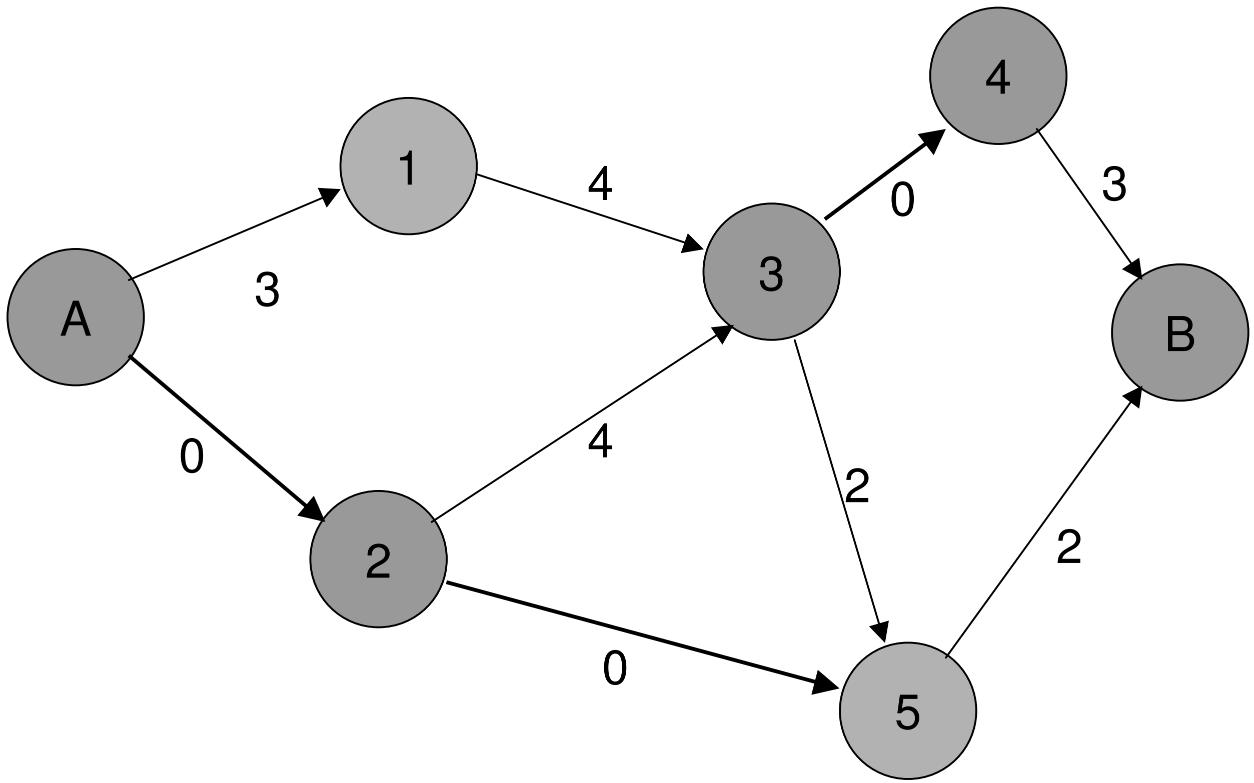
An example

- Current flow = 1



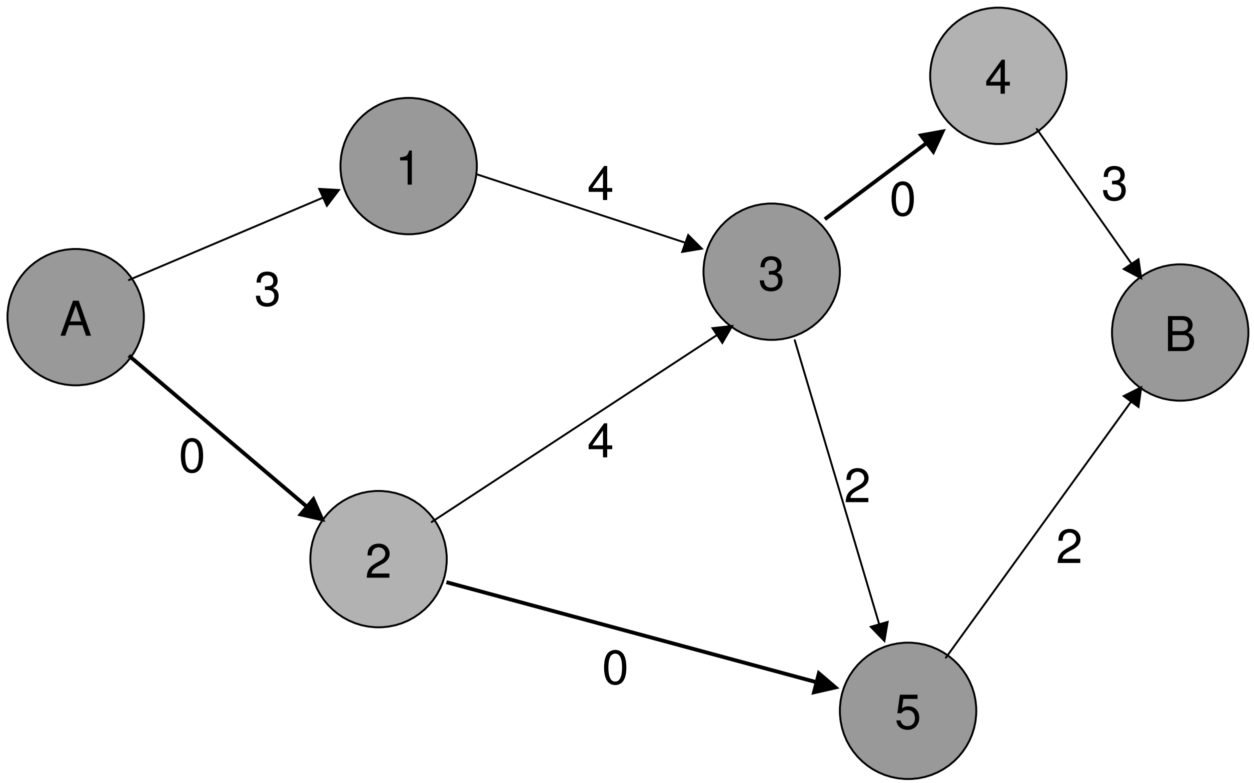
An example

- Current flow = 3



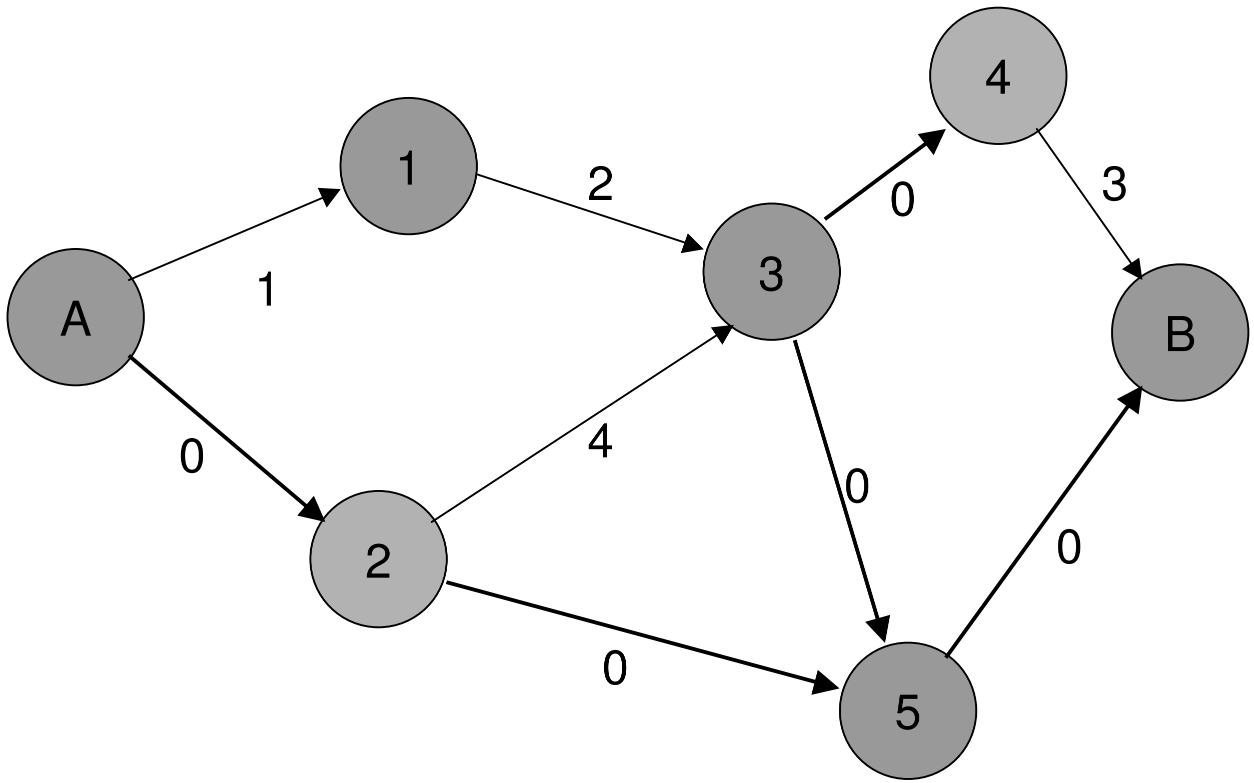
An example

- Current flow = 3



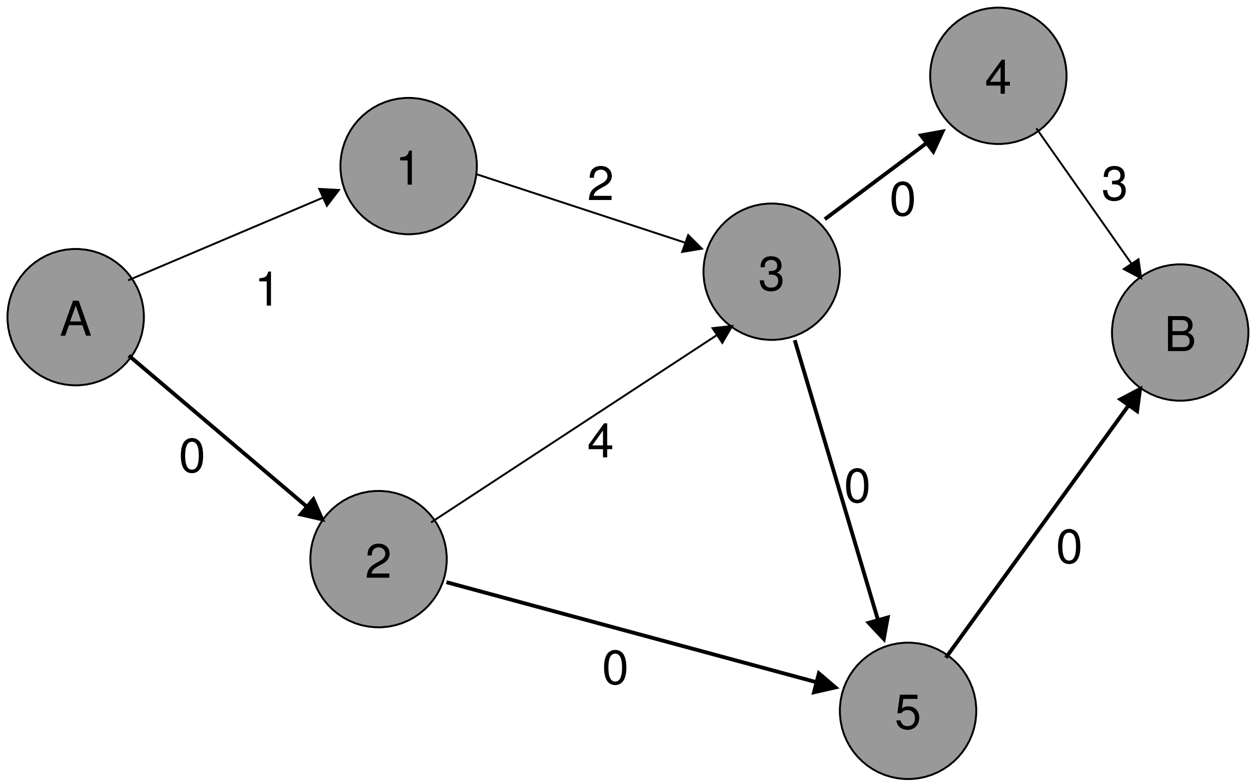
An example

- Current flow = 5



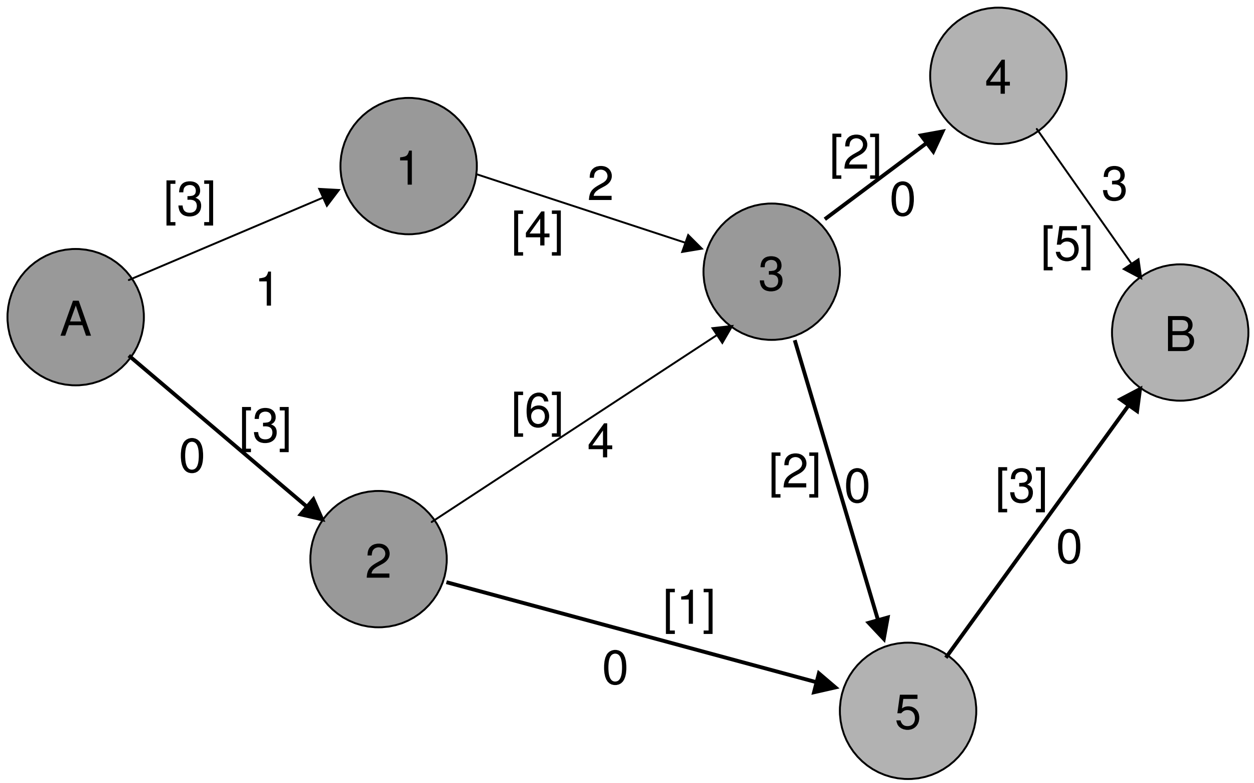
An example

- Total flow = 5



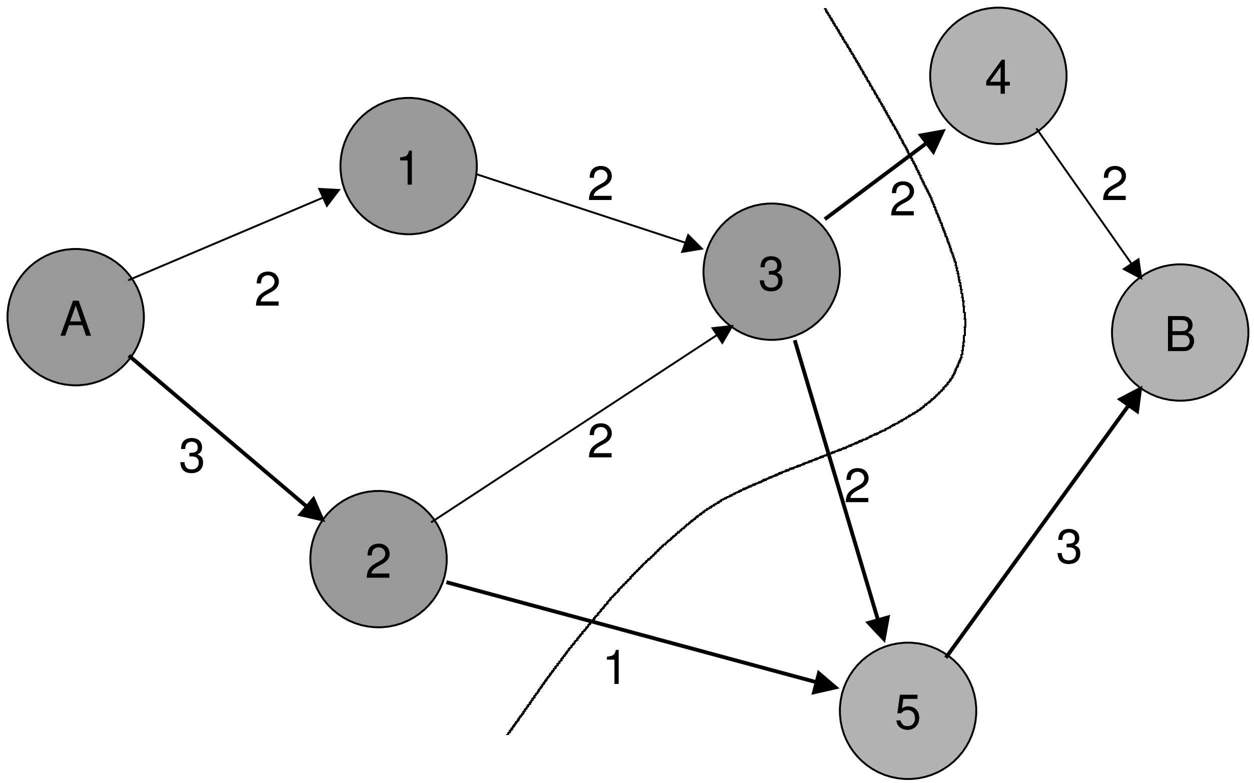
An example

- To find a minimal cut



An example

- Minimal cut = 1 + 2 + 2 = 5

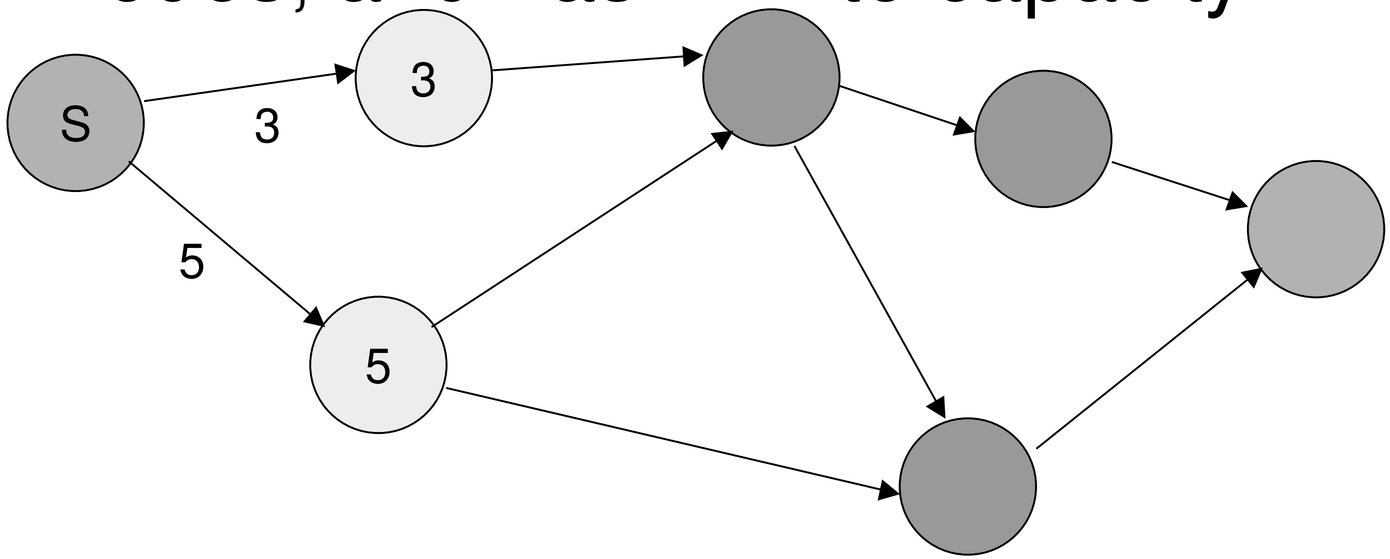


Variations on Network Flow

- Undirected graphs – give the edge the capacity in both directions, then increase or decrease this capacity as flow changes, as before.

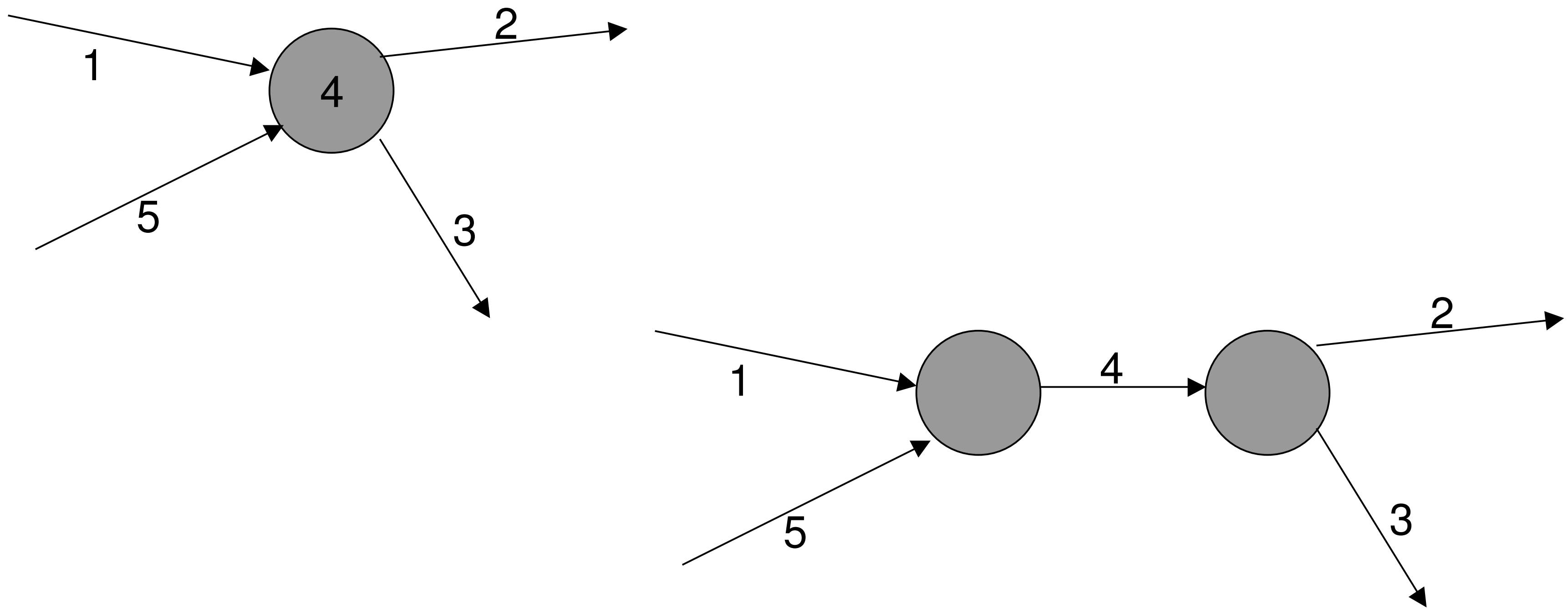
Variations on Network Flow

- Multiple sources / sinks – create a "supersource" or "supersink" which connects directly to each of these nodes, and has infinite capacity



Variations on Network Flow

- Node capacity – split the node into an "in" node, an "out" node and an edge



Conclusion

- Network flow problems can come in a number of forms, usually relating to some commodity being moved from suppliers to where it is needed.
- Also look for minimal cut problems, which can be solved easily with graph theory.